ΛΥΣΕΙΣ ΔΙΑΓΩΝΙΣΜΑΤΟΣ Γ' ΛΥΚΕΙΟΥ 4/4/2021

Θέμα Α   A1 – δ   A2 – α   A3 – β   A4 – γ   A5 ΣΛΛΛΣ

Θέμα Β

$\boxed{B1 \mid I-\alpha}$ $\quad \bar{P}_1 = \dfrac{V_{εν}^2}{R_1}, \quad \bar{P}_2 = \dfrac{V_{εν}^2}{R_2}, \quad \bar{P} = \dfrac{V_{εν}^2}{R_{ολ}} = \dfrac{V_{εν}^2}{R_1+R_2} = \dfrac{V_{εν}^2}{\dfrac{V_{εν}^2}{\bar{P}_1} + \dfrac{V_{εν}^2}{\bar{P}_2}} = \dfrac{1}{\dfrac{1}{\bar{P}_1} + \dfrac{1}{\bar{P}_2}}$

$$\Rightarrow \bar{P} = \frac{1}{\dfrac{\bar{P}_2+\bar{P}_1}{\bar{P}_1 \cdot \bar{P}_2}} \Rightarrow \boxed{\bar{P} = \frac{\bar{P}_1 \cdot \bar{P}_2}{\bar{P}_1 + \bar{P}_2}}$$

$\boxed{B_1 \mid II-\gamma}$ $\quad \bar{P}_1 = P_{2max} \Rightarrow \dfrac{V_{εν}^2}{R_1} = \dfrac{V^2}{R_2} \Rightarrow \dfrac{V^2}{2R_1} = \dfrac{V^2}{R_2} \Rightarrow R_2 = 2R_1$

$$R_{ολ} = \frac{R_1 R_2}{R_1+R_2} = \frac{2R_1^2}{3R_1} \Rightarrow R_{ολ} = \frac{2}{3}R_1$$

$$\bar{P} = \frac{V_{εν}^2}{R_{ολ}} = \frac{V_{εν}^2}{\frac{2}{3}R_1} = \frac{3}{2}\frac{V_{εν}^2}{R_1} = \frac{3}{2}\bar{P}_1 \Rightarrow \boxed{\bar{P} = \frac{3}{2}\bar{P}_1}$$

$\boxed{B2-\beta}$ $\quad$ ΑΔΟ : $\vec{P}_{πριν} = \vec{P}_{μετα} \Rightarrow P_m = P_k \Rightarrow mv = (m+M)v_k$

$$\Rightarrow mv = 4m v_k \Rightarrow v_k = v/4.$$

$$K_{πριν} = \frac{1}{2}mv^2 \quad K_{μετα} = \frac{1}{2}(m+M)v_k^2 = \frac{1}{2}4m\frac{v^2}{16} = \frac{1}{4}\frac{1}{2}mv^2 \Rightarrow K_{μετα} = \frac{1}{4}K_{πριν}$$

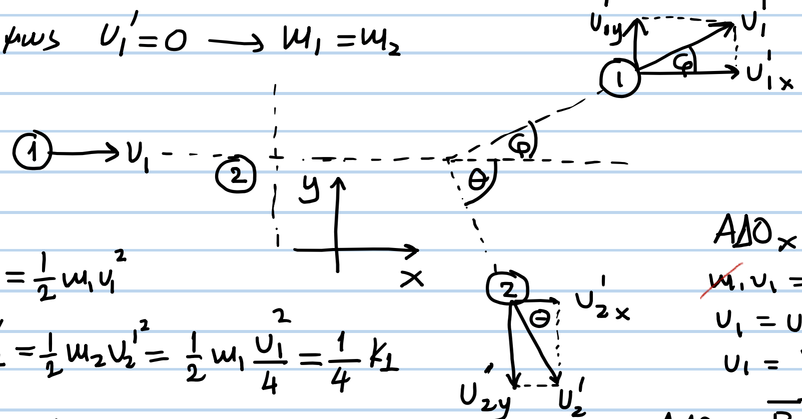
$$E_1 = K_{πριν} - K_{μετα} = K_{πριν} - \frac{1}{4}K_{πριν} \Rightarrow E_1 = \frac{3}{4}K_{πριν}$$

$$|W_T| = Q_{τριβης} = E_2 = K_{μετα} = \frac{1}{4}K_{πριν}$$

Άρα $\dfrac{E_2}{E_1} = \dfrac{1/4\, K_{πριν}}{3/4\, K_{πριν}} \Rightarrow \boxed{\dfrac{E_2}{E_1} = \dfrac{1}{3}}$

$\boxed{B3-\alpha}$ $\quad K_1 = K_2' \; (K_1'=0)$ για την κεντρική κρούση $\rightarrow v_1' = \dfrac{m_1-m_2}{m_1+m_2}v_1$

όμως $v_1' = 0 \longrightarrow m_1 = m_2$



$\varphi + \theta = 90^\circ$ $\quad \boxed{\text{με απόδειξη}}$

$\theta = 60^\circ$

ΑΔΟ$_x$: $\vec{P}_{xπριν} = \vec{P}_{xμετα}$

$\cancel{m_1}v_1 = \cancel{m_1}v_{1x}' + \cancel{m_2}v_{2x}'$

$v_1 = v_1'\,συνφ + v_2'\,συνθ$

$v_1 = \frac{\sqrt{3}}{2}v_1' + \frac{1}{2}v_2'$ (α)

$K_1 = \frac{1}{2}m_1 v_1^2$

$K_2' = \frac{1}{2}m_2 v_2'^2 = \frac{1}{2}m_1\frac{v_1^2}{4} = \frac{1}{4}K_1$

ΑΔΟ$_y$: $\vec{P}_{yπριν} = \vec{P}_{yμετα} \Rightarrow 0 = m_1 v_{1y}' - m_2 v_{2y}'$

$\Rightarrow \cancel{m_1}v_1'\,ημφ = \cancel{m_2}v_2'\,ημθ \Rightarrow \frac{1}{2}v_1' = \frac{\sqrt{3}}{2}v_2'$

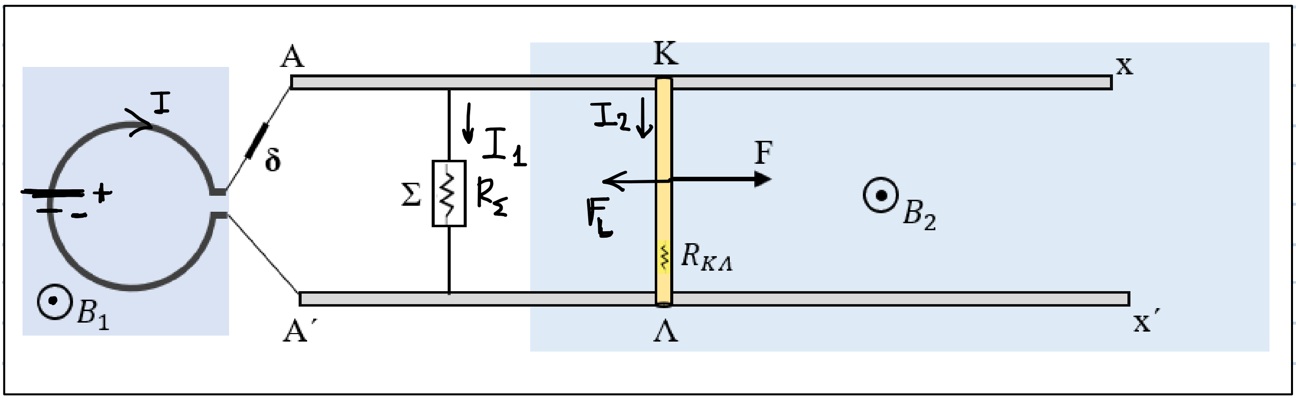
$\Pi = \dfrac{K_2'}{K_1}100\% = \dfrac{1/4\,K_1}{K_1}100\%$

$\Rightarrow v_1' = \sqrt{3}\,v_2'$ (β)

$\boxed{\Pi = 25\%}$

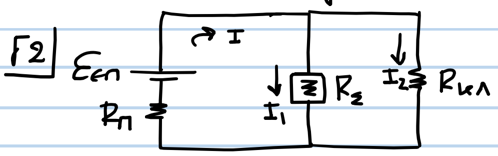
(α) $\overset{(β)}{\Longrightarrow} v_1 = \frac{\sqrt{3}}{2}\sqrt{3}v_2' + \frac{1}{2}v_2' \Rightarrow v_1 = 2v_2' \Rightarrow v_2' = \frac{v_1}{2}$

Θέμα Γ



Γ1 $\Sigma F = 0 \rightarrow \vec{F}_L$ αριστερά, ρεύμα στον αγωγό από το $K \rightarrow Λ$ άρα το ρεύμα I στο κυκλικό πλαίσιο με δεξιόστροφη φορά οπότε $\vec{B}_{επ}$ ⊗ άρα $\vec{B}_1$ ⊙ (λόγω Lenz)

Γ2



για συσκευή $P_K = \frac{V_K^2}{R_Σ} \Rightarrow R_Σ = \frac{V_K^2}{P_K} \Rightarrow R_Σ = 3\Omega$

$I_K = \frac{V_K}{R_Σ} = 2A$

$R_{1Σ} = \frac{R_Σ R_{KΛ}}{R_Σ + R_{KΛ}} = 1,2\Omega$ άρα $R_{ολ} = R_{1Σ} + R_π \Rightarrow R_{ολ} = 3\Omega$

$\Sigma F = 0 \Rightarrow F = F_L = B_2 I_2 \ell \Rightarrow I_2 = F/B_2\ell \Rightarrow I_2 = 2A$

$V_{R_Σ} = V_{R_{KΛ}} \Rightarrow I_1 R_Σ = I_2 R_{KΛ} \Rightarrow 3I_1 = 2I_2 \Rightarrow I_1 = \frac{2}{3} I_2 \Rightarrow I_1 = \frac{4}{3} A$

Επειδή $I_1 = 4/3\,A < I_K = 2A$ η συσκευή υπολειτουργεί.

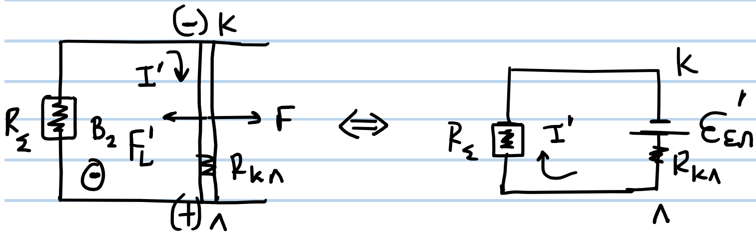
$I = I_1 + I_2 \Rightarrow I = 10/3\,A$

$I = \frac{Ε_{επ}}{R_{ολ}} \Rightarrow Ε_{επ} = I R_{ολ} \Rightarrow N\frac{\Delta\Phi}{\Delta t} = I \cdot R_{ολ} \Rightarrow \boxed{\frac{\Delta\Phi}{\Delta t} = 0,5\,Wb/s}$

Γ3 δ-ανοικτός $I_1 = I_2 = I = 0$ οπότε λόγω $\vec{F}$ ο αγωγός αρχίζει να κινείται δεξιά μέσα στο ΟΜΠ $\vec{B}_2$ οπότε $Ε'_{επ} = \frac{\Delta\Phi_2}{\Delta t} = \frac{B_2 \Delta x \ell}{\Delta t} = B_2 υ \ell$.

$υ \uparrow\ Ε_{επ} \uparrow\ I' \uparrow\ F'_L \uparrow\ \Sigma F' = F - F'_L \downarrow$ όταν $\Sigma F' = 0 \rightarrow υ_{ορ}$

$\Sigma F' = 0 \Rightarrow F = F'_L \Rightarrow F = B_2 I' \ell \Rightarrow F = B_2 \frac{B_2 υ_{ορ} \ell}{R'_{ολ}} \ell \Rightarrow υ_{ορ} = \frac{F \cdot R'_{ολ}}{B^2 \ell^2}$



όπου $R'_{ολ} = R_Σ + R_{KΛ} = 5\Omega$

$\boxed{υ_{ορ} = 10\,m/s}$

Γ4 όταν $\upsilon = \upsilon_{op}$ $I' = \frac{E_{\varepsilon\pi}'}{R_{o\lambda}'} = \frac{B_2 \upsilon_{op} \ell}{R_{o\lambda}'} \Rightarrow I' = 2A = I_k$ λειτουργεί κανονικά

Γ5 δ-κλειστός $I_1 = \sigma\tau\alpha\theta \Rightarrow I_1 = \frac{\Delta q}{\Delta t} \Rightarrow \Delta q = I_1 \cdot \Delta t \Rightarrow \Delta q = 3C$

για την κίνηση του αγωγού $\Delta q = \frac{\Delta \Phi_2}{R_{o\lambda}'} = \frac{B_2 \ell \cdot \Delta x}{R_{o\lambda}'} \Rightarrow \Delta x = \frac{\Delta q \cdot R_{o\lambda}'}{B_2 \cdot \ell}$

$\Rightarrow \Delta x = 15\,m$

ΑΔΕ : $W_F = K_{\tau\varepsilon\lambda} + Q_{R_{o\lambda}'} \Rightarrow F \cdot \Delta x = \frac{1}{2} m \upsilon_{op}^2 + Q_{R_{o\lambda}'}' \Rightarrow \boxed{Q_{R_{o\lambda}}' = 10J}$

Θέμα Δ

Δ1 $V_1 = y_1 A_\delta = 0{,}72\,m^3$ $\Pi_{\alpha\nu\tau} = \frac{V_1}{\Delta t_1} \Rightarrow \boxed{\Pi_{\alpha\nu\tau} = 8 \cdot 10^{-4}\ m^3/s}$

$\Delta t_1 = 15 min = 900\,sec$

Δ2 ΘΜΚΕ $K_{\tau\varepsilon\lambda_B} - \cancel{K_{\alpha\rho\chi_A}} = W_{\beta\alpha\rho o\upsilon\varsigma} + W_{\alpha\nu\tau} + W_{\pi\iota\varepsilon\sigma\eta\varsigma\ \rho\varepsilon\upsilon\sigma\tau o\acute{\upsilon}}$

$\frac{1}{2} \Delta m \cdot \upsilon^2 - 0 = -\Delta m g h + W_{\alpha\nu\tau} + (P_{A\alpha\tau\mu} - P_{B\alpha\tau\mu}) \Delta V$

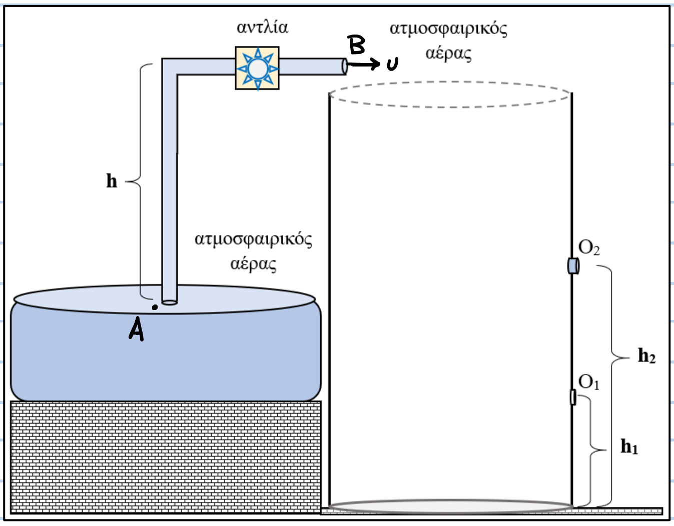
$\frac{1}{2} \rho \Delta V \upsilon^2 + \rho \Delta V \cdot g h = W_{\alpha\nu\tau}$

$\Delta V = V_1 = 0{,}72\,m^3$, $\Pi_{\alpha\nu\tau} = A_{\alpha\nu\tau} \cdot \upsilon$

$\Rightarrow \upsilon = 2\,m/s$

$W_{\alpha\nu\tau} = \frac{1}{2} \rho V_1 \upsilon^2 + \rho V_1 g h$

$\boxed{W_{\alpha\nu\tau} = 8640\ J}$



Δ3 $\Pi_{\alpha\nu\tau} = \Pi_{(O_1)} = A \cdot \upsilon_1$

$\Rightarrow \upsilon_1 = \frac{\Pi_{\alpha\nu\tau}}{A} \Rightarrow \upsilon_1 = 4\,m/s.$

Bernoulli από την επιφάνεια του νερού στο δοχείο Δ1 ($H_1 = \sigma\tau\alpha\theta$) στην οπή O_1

$P_{\alpha\tau\mu} + \rho g H_1 + 0 = P_{\alpha\tau\mu} + \rho g h_1 + \frac{1}{2} \rho \upsilon_1^2 \Rightarrow H_1 = h_1 + \frac{\upsilon_1^2}{2g} \Rightarrow \boxed{H_1 = 1{,}6\,m}$

Δ4 Ισχύει $\Pi_{\alpha\nu\tau} = \Pi_{(O_1)} + \frac{\Delta V_{\Delta_1}}{\Delta t} \Rightarrow \Pi_{\alpha\nu\tau} = A \cdot \upsilon' + \frac{1}{\rho} \frac{\Delta m}{\Delta t}$

$\Rightarrow 8 \cdot 10^{-4} = 2 \cdot 10^{-4} \cdot \upsilon' + 5 \cdot 10^{-4} \Rightarrow \boxed{\upsilon' = 1{,}5\,m/s}$

Δ5 Νέα παροχή αντλίας: $\Pi_{\alpha\nu\tau}' = \Pi_{\alpha\nu\tau} + 150\%\,\Pi_{\alpha\nu\tau} = \Pi_{\alpha\nu\tau} + 1{,}5\,\Pi_{\alpha\nu\tau}$

$\Pi_{\alpha\nu\tau}' = 2{,}5\,\Pi_{\alpha\nu\tau} = 2{,}5 \cdot 8 \cdot 10^{-4} \Rightarrow \Pi_{\alpha\nu\tau}' = 20 \cdot 10^{-4}\ m^3/s.$

αρχική μέγιστη οριζόντια απόσταση από οπή O_1 : $S_1 = \upsilon_1' \cdot t_{1\varepsilon\delta}$

όπου $h_1 = \frac{1}{2} g t_{1\varepsilon\delta}^2 \Rightarrow t_{1\varepsilon\delta} = \sqrt{\frac{2h_1}{g}} = 0{,}4\,sec \rightsquigarrow S_1 = 1{,}6\,m.$

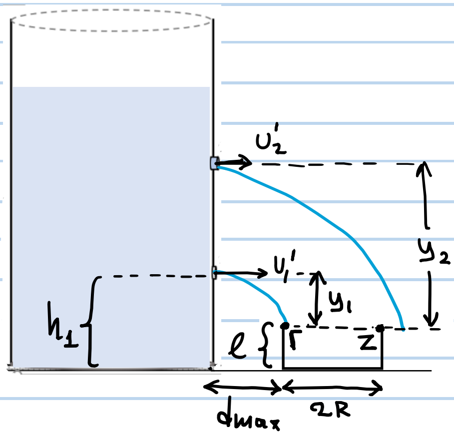
νέα μέγιστη οριζόντια απόσταση από οπή O_1 : $S_1' = S_1 + 0{,}8m \Rightarrow S_1' = 2{,}4m$

ισχύει $S_1' = u_1' t_{\iota\varepsilon\varsigma} \Rightarrow u_1' = \dfrac{S_1'}{t_{\iota\varepsilon\varsigma}} \Rightarrow u_1' = 6\ m/s$.

$H_2 = \sigma\tau\alpha\theta$: $\Pi_{\alpha\nu\tau}' = \Pi_{(O_2)} + \Pi_{(O_1)} \Rightarrow \Pi_{\alpha\nu\tau}' = A_2 u_2' + A_1 u_1'$

$$\Rightarrow 20 \cdot 10^{-4} = 2 \cdot 10^{-4} \cdot 6 + 2 \cdot 10^{-4} \cdot u_2' \Rightarrow \boxed{u_2' = 4\ m/s}$$

Δ6



Σημείο Γ του δοχείου Δ_2

$x_1 = d_{max}$

$y_1 = h_1 - \ell \Rightarrow y_1 = 0{,}2m$

Ισχύει $x_1 = u_1' t \Rightarrow t = \dfrac{x_1}{u_1'}$

$$y_1 = \frac{1}{2} g t^2 = \frac{1}{2} g \left(\frac{x_1}{u_1'}\right)^2$$

$$\Rightarrow y_1 = \frac{g x_1^2}{2u_1'^2} \Rightarrow x_1^2 = \frac{2 y_1 u_1'^2}{g}$$

$$\Rightarrow x_1^2 = 1{,}44 \Rightarrow x_1 = 1{,}2m = d_{max}$$

Το σημείο Z του δοχείου : $x_2 = d_{max} + 2R \Rightarrow x_2 = 1{,}6m$.

$y_2 = h_2 - \ell \Rightarrow y_2 = 1{,}2m$

Εξετάζουμε αν η φλέβα του νερού από την οπή O_2 εισέρχεται μέσα στο δοχείο Δ_2.

$$y_2 = \frac{g x_2'^2}{2u_2'^2} \Rightarrow x_2'^2 = \frac{2 y_2 u_2'^2}{g} \Rightarrow x_2' = \sqrt{3{,}84}\ m = 1{,}96m$$

Όμως $x_2 = 1{,}6m < x_2' = 1{,}96m$ άρα δεν εισέρχεται

Το γέμισμα του δοχείου Δ_2 γίνεται μόνο από το νερό που εξέρχεται από την οπή O_1

$$V_{\Delta_2} = \ell \pi R^2 = 0{,}6 \cdot \pi \cdot 0{,}04\ m^3 \Rightarrow V_{\Delta_2} = 24\pi \cdot 10^{-3}\ m^3$$

$$\Pi_{(O_1)}' = \frac{V_{\Delta_2}}{\Delta t} \Rightarrow A \cdot u_1' = \frac{V_{\Delta_2}}{\Delta t} \Rightarrow \Delta t = \frac{V_{\Delta_2}}{A \cdot u_1'} = \frac{24\pi \cdot 10^{-3}}{12 \cdot 10^{-4}}$$

$$\Rightarrow \boxed{\Delta t = 20\pi\ sec = 62{,}8\ sec}$$